Optimum Configuration for a 10-Passenger Business Turbofan Jet Airplane

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The optimum external configuration for an airplane to meet specific design and performance requirements is determined by use of an analytical method that considers systematic variations of wing sweep, thickness ratio, fuselage fineness ratio, cruising speed, and design weight. Relationships among the various structural, weight, and aerodynamic parameters used in the method are derived. Evaluation of the parameters is then based on limits imposed by current experience and assumptions. The specific design objectives in this study were to minimize takeoff weight and maximize the cruising speed for a specific range with a designated powerplant. The optimum configuration resulting from this study has a 36-deg wing sweep and 14.6% wing section thickness, a ramp weight of 21,575 lb, and a range of 2500 n.mi. with 10 passengers. Besides providing an optimum design and an analytical method for its determination, the method and a number of the curves can be used to determine configurations which will meet other specific performance requirements.

Nomenclature

A = ratio of landing weight to takeoff weight $= W_F / W_0$
A_R, A_V = values of A computed by Eqs. (23) and (24), respectively
$\mathcal{R}$ = aspect ratio $= b^2/S$
b = airplane wing span measured normal to XZ plane of airplane, ft
$C_{D_{W+T}}$ = drag coefficient of wing and tail surfaces based on wing area
$C_{D_{R0}}, C_{D_{\pi F}}$ = drag coefficient of fuselage, nacelles, and windshield, based on wing area and maximum frontal area of fuselage, respectively
C_{D_N} = drag coefficient of nacelles based on frontal area of fuselage
C_{D_C} = drag coefficient of windshield based on frontal area of fuselage
d' = mean maximum fuselage diameter $= \sqrt{\text{width} \times \text{height}}$, ft
e = Oswald's airplane efficiency factor
F = maximum fuselage frontal area, ft^2
f = fuselage fineness ratio $= l/d'$
K = induced drag factor $= 1/\pi e$
K_{TD} = thickness ratio correction factor for drag $= [C_{D_W}$ (with thickness ratio X) / C_{D_W} (with thickness ratio 12%)$]^{1/2}$
$(L/D)_{\max}$ = airplane maximum lift over drag ratio
l = maximum fuselage length, ft
M = Mach number
q = dynamic pressure, psf
R = range, n.mi.
S = wing area, ft^2
TSFC = thrust specific fuel consumption = (lb/h of fuel)/(lb of thrust)
T_0 = uninstalled static jet thrust, standard conditions at sea level at takeoff, lb
T_r = thrust required, lb
t/c = wing thickness ratio for chord parallel to XZ plane
V = true airspeed, knots
V_i = indicated airspeed, knots
Vol, Vol$_0$ = volume and total usable volume, ft^3
W = gross weight, lb
λ = taper ratio = tip chord/root chord
Λ = sweep angle of wing quarter chord line, deg
ρ_0 = 0.002378 slug/ft^3
σ = air density ratio $= \rho/\rho_0$

Subscripts

ave = average
c = cruising
eng = engine
F = landing
fus = fuselage
L.G. = landing gear
nac = nacelle
R = reference
w = wing
0 = design
36 = at altitude of 36,089 ft and above

Introduction

THIS paper describes a method of determining the optimum configuration of a business turbofan airplane, that is, to establish such values of aspect ratio, wing thickness, sweepback, fuselage size, and fineness ratio when used with a specific powerplant so as to minimize the maximum weight and maximize the cruising speed for a specific range.

When designing business jet aircraft for long-range cruise, a major design problem is that of providing space for the large quantity of fuel required to accomplish the mission. This is a difficult task because if high cruising speeds are desired, conventional airfoil wings must have either small thickness ratios, sweepback, or low aspect ratios. Since fuel will have to be contained in both the fuselage and wing, it is necessary to analyze the configuration and to balance the low drag of a high-speed wing, with its small volume, and the relatively high drag of a large fuselage, with its large volume, so that the drag of the combination is as low as practical for the requirements of the mission. The above statement represents the problem at hand.

Analysis

The contemplated analysis can be carried out by one of two methods. The first method is to establish a number of airplane designs and analyze the performance of each. From a study of

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the results, the optimum configuration possibly can be estimated. The second method is to establish relationships between the various structural, weight, and aerodynamic parameters and substitute these in the range equation, then solve the range equation for the combination of parameters. Evaluation of the parameters is then based on limits imposed by present practical experiences and assumptions.

The second method is used herein. It is the more desirable one because the effects of the various parameters can be better traced, if later modifications are desired, and are less obscured by unknown factors, which are present in every design, than by the use of the first method. The present solution is based on the work of Ref. 1, where the optimum configuration of a long-range jet fighter airplane was determined. Necessary changes to relationships in the equations of Ref. 1 to meet the current mission requirements will be described.

Design Considerations

The business jet airplane will have a useful load of 2 pilots, 10 passengers, their baggage, and fuel. Only the fuel load will vary from beginning to end of flight. The propulsion system will consist of two GA TFE 731-3 engines mounted separately in nacelles mounted on the fuselage. This will permit utilization of more of the fuselage for fuel stowage and will reduce the induction and tailpipe losses of the installation. The airplane will have a conventional configuration; that is, it will consist of wing, fuselage, engine nacelles, and tail surfaces. Stability considerations will not be used to establish relationships between tail and wing areas or tail length, for it is assumed that there will be sufficient latitude in the optimum configuration to establish satisfactory stability and control.

The mission to be flown in based on market surveys and meets the following requirements:

1) a maximum range of 2500 n.mi. with 45-min fuel reserve onboard with 10 passengers,

2) a balanced field length for takeoff not exceeding 5000 ft at maximum takeoff weight,

3) a cruising altitude of 40,000 ft and above, and

4) an approach speed at maximum landing weight not exceeding 130 knots.

Aerodynamic Data

The main portion of the mission is the cruise portion, and the design is mainly predicted on the analytical solution here. The other segments, such as takeoff, climb, descent, and landing, are covered subsequently. Breguet's range equation, as adapted for jet aircraft and for flight at altitudes of 36,089 ft and higher, is[2]

$$R = [V_c/(\mathrm{TSFC})(L/D)]_{\max} \ln W_0/W_F \tag{1}$$

For this condition, the optimum cruising speed will remain constant, as the following brief proof shows. Let the optimum cruising speed be expressed as

$$V_c = \sqrt{\frac{3}{4}}\sqrt{\frac{2}{C_{D_p}S}\frac{T_A}{\rho}}$$

for a parabolic airplane polar in incompressible flow, where T_A is the jet thrust available; ρ is the air density; and C_{D_p} is the total parasite drag. The values of T_A and ρ at 36,089 ft altitude will be taken as the reference values. From Ref. 2, $T_A = T_{36}(\sigma/\sigma_{36})$ for constant true airspeeds and higher altitudes, and $\rho = \rho_{36}(\sigma/\sigma_{36})$ by definition; then $T_A/\rho = T_{36}/\rho_{36}$, and the optimum cruising speed is constant for all altitudes above 36,089 ft up to drag divergence.

When Eq. (1) is rearranged thus,

$$[R(\mathrm{TSFC}_{36})/V_c](D/L)_{\min} = \ln W_0/W_F \tag{2}$$

$(D/L)_{\min}$ is proportional to $\ln W_0/W_F$ if $R(\mathrm{TSFC}_{36})/V_c$ is fixed. If a true average value of thrust required occurring during the flight is used, then the expression $R(\mathrm{TSFC}_{36})T_{r_{\mathrm{ave}}}/V_c$ is the fuel used for a certain duration of flight, R/V_c. Dividing by a reference weight, such as the design weight W_0, then

$$\frac{R(\mathrm{TSFC}_{36})}{V_c}\frac{T_{r_{\mathrm{ave}}}}{W_0} = \frac{W_{\mathrm{fuel}}}{W_0} = 1 - \frac{W_F}{W_0} \tag{3}$$

where $W_{\mathrm{fuel}} = W_0 - W_F$. In Eq. (3), $T_{r_{\mathrm{ave}}}/W_0$ is proportional to $1 - W_F/W_0$ if $R(\mathrm{TSFC}_{36})/V_c$ is fixed.

It is obvious that Eqs. (2) and (3) are of the same form and either could be used to determine the unknown parameters. In Ref. 3 the relationship between $T_{r_{\mathrm{ave}}}/W_0$ and $1 - W_F/W_0$ has been established; thus the range equation in the form of Eq. (3) will be used to establish the optimum configuration relationships. From Ref. 3,

$$\frac{T_{r_{\mathrm{ave}}}}{W_0} = \phi\sqrt{\frac{1+A+A^2}{3}} \tag{4}$$

where

$$\phi = 2\sqrt{\frac{C_{D_{W+T}}K}{\mathcal{R}}} + \frac{C_{D_{RO}}}{W_R/S_R q} \tag{5}$$

and

$$A = \frac{W_F}{W_0} \tag{6}$$

Combining Eqs. (3-6) results in

$$\frac{R(\mathrm{TSFC}_{36})}{V_c}\left[2\sqrt{\frac{C_{D_{W+T}}K}{\mathcal{R}}} + \frac{C_{D_{RO}}}{W_R/S_R q}\right] = (1-A)\Big/\sqrt{\frac{1+A+A^2}{3}} \tag{7}$$

The purpose of the succeeding sections is to obtain consistent relationships between the parameters in Eq. (7), other than those established in Ref. 1 and used without change.

Evaluation of Eq. (7) Terms

Aerodynamic Considerations

From Ref. 1,

$$C_{D_{W+T}} = C_{D_W}(1+0.35)$$
$$= 0.0060(1+0.35)K_{TD}^2$$
$$= 0.0081K_{TD}^2 \tag{8}$$

K_{TD} is given in Fig. 1, which is from Ref. 1.

The value of K is by definition $1/\pi e$. The value of e for a carefully designed planform will be about 0.8.

By definition

$$C_{D_{RO}} = C_{D\pi F}F/S \tag{9}$$

where $C_{D\pi F}$ is the drag coefficient for fuselage, windshield, and nacelles based on fuselage frontal area F. As the nacelles containing the turbofan engines will be fixed in size, their drag can be established independent of fuselage frontal area. A properly designed nacelle based on the recommendations of Ref. 4 will have a drag coefficient of 0.0900 based on its own frontal area. The frontal area of the two nacelles based on the maximum diameter is 16.0 ft^2. Therefore the nacelle drag coefficient based on fuselage frontal area is $C_{D_N} = 0.0900 \times 16.0/F$.

The drag coefficient for a well-designed windshield is 0.0038 (Ref. 5). The frontal area is estimated to be 7.0 ft^2. Therefore the drag coefficient based on fuselage frontal area is $C_{D_C} = 0.0038 \times 7.0/F$. From Ref. 1, the full-scale Reynold's number fuselage drag coefficient based on frontal area is

$$C_{D\pi F} = -0.0218 + 0.011f \qquad (10)$$

The drag coefficient of the windshield, nacelles, and fuselage based on the fuselage frontal area is

$$C_{D\pi F} = 1.466/F - 0.0218 + 0.011f \qquad (11)$$

and

$$C_{D_{RO}} = \left(\frac{1.466}{F} - 0.0218 + 0.011f\right)F/S \qquad (12)$$

Relation of Best Cruising Speed to Wing Thickness Ratio and Fuselage Fineness Ratio

In order to obtain a systematic variation of wing thickness ratio, sweep, and fuselage fineness ratio with desired cruising speed for insertion in Eqs. (23) and (24), a study of the range performance of a number of airplanes was made. The basic range equation, Eq. (1), by proper substitution, is shown to depend on the product of the flight Mach number and the maximum lift over drag ratio, $M(L/D)_{max}$, for compressible flow conditions. With the other terms in the equation constant, it is seen that the range will increase with an increase in the flight Mach number as long as compressibility does not reduce the value of the maximum lift over drag ratio. Examination of experimentally obtained curves for $M(L/D)_{max}$, and total airplane drag coefficient against Mach number for a number of airplane designs in Ref. 1, indicated that the Mach number for maximum $M(L/D)_{max}$ corresponded very closely to the Mach number for drag rise (within a 0.01-0.02 change in Mach number). From that it is concluded in Ref. 1 that, with the assumption made regarding engine output, the maximum range computed for the speed at the drag rise Mach number is equal to that computed for the speed at the Mach number for maximum $M(L/D)_{max}$.

The components which contribute most to the airplane drag are the wings, fuselage, and nacelles, and, as a result, their drag rise Mach number should have a large effect on the speed for maximum range. In Ref. 1, it was concluded that both the wing and fuselage should have the same drag rise Mach number and that it will also represent the assumed best cruising speed Mach number. Experimental data from high-speed wind tunnel tests were used to establish the drag rise Mach number in relation to thickness and sweep for wings and fineness ratio for fuselages. The results[1] are shown in Figs. 2 and 3.

Engine Performance Considerations

The TFE 731-3 engine thrust and specific fuel consumption (TSFC) are obtained from the engine manufacturer's specification and have been summarized in Table 1. For the cruise condition, TSFC at 41,000 ft altitude and $M = 0.8$ is 0.829 for the uninstalled engine. At the low thrusts considered, the variation of thrust with speed is negligible and a constant value is sufficiently accurate at all speeds. The specific fuel consumption will also be approximately constant with airspeed for small changes in airspeed. The use of uninstalled values, for these preliminary studies, gives thrust values which are about 4% too high and specific fuel consumptions which are about 2% too high.

Range Considerations

The maximum range will be 2500 n.mi., based on design considerations, and includes the accelerate and climb distances and the descent distance. For the initial calculations, an allowance of 200 n.mi. for climb and descent distances is

Table 1 GA TFE 731-3 turbofan engine dimensions and performance

Dimensions			
Maximum diameter	34.20 in.		
	39.47 in., including gear accessory box		
Maximum length	49.73 in.		
Volume	16.6 ft^3		
Frontal area	7.4 ft^2		
Weight	735 lb dry		
Uninstalled Performance			
Power setting	Thrust, lb	C	Engine inlet airflow, lb/s
Takeoff ($M = 0$, sea level)	3700	0.506	120
Climb ($M = 0.2$, sea level)	3050	0.616	123
Maximum cruise			
at sea level ($M = 0.6$)	2000	0.970	138
at 36,089 ($M = 0.8$)	990	0.818	50.5
at 40,000 ($M = 0.8$)	817	0.819	44
at 50,000 ($M = 0.8$)	492	0.856	26

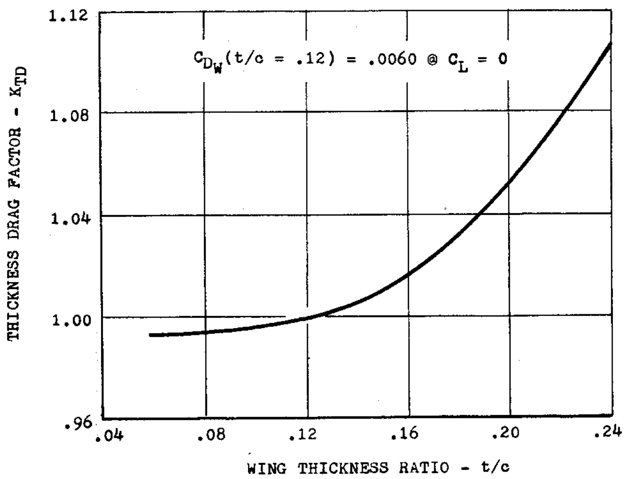

Fig. 1 Wing relative drag increase due to thickness.

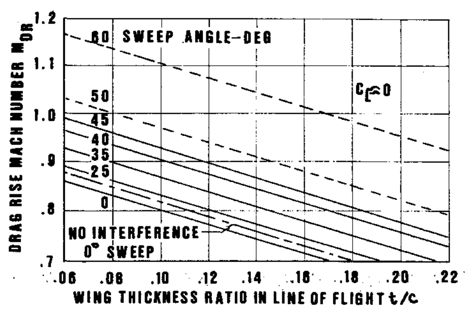

Fig. 2 Drag rise Mach number for wings of various thickness ratios and sweep.

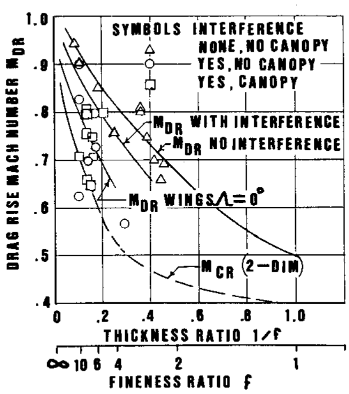

Fig. 3 Drag rise Mach number for streamline bodies of various fineness ratios.

included. Because the dynamic pressure will change with altitude if the cruise speed is kept constant, the simplification that, for altitudes of 36,089 ft and above, the constant altitude range is very close to the range flown at increasing altitude is used. Therefore the dynamic pressure value required in Eq. (7) is now a function of cruising speed only.

It is seen that the parameters to be substituted into Eq. (7) are now more specific in their definition of the airplane configuration than the term $M(L/D)_{\max}$. Relationships between certain of the terms can now be established based on current design practices.

Volume Considerations

An airplane must have space to carry a certain amount of equipment and useful load irrespective of the range it has. Thus space is required for the engines, passengers, baggage, pilots, avionics equipment, fuel, etc. For this size airplane, the fuel will be carried in the fuselage and the wings.

Let the total usable volume of an airplane be made up of the wing volume, fuselage volume, and engine nacelles volume. The usable volume is defined as that volume that can be efficiently used for storage of any of the airplane basic equipment, fuel, passengers, pilots, etc. Thus the tail surfaces are normally eliminated as storage space except for items such as radio antennas, pitot head plus tubing, and the control surfaces' operating mechanisms. Thus

$$\mathrm{Vol}_W + \mathrm{Vol}_{\mathrm{fus}} + \mathrm{Vol}_{\mathrm{nac}} = \mathrm{Vol}_0 \tag{13}$$

Applying the definition of usable volume and accounting for all the items contained in the airplane, rearrange the equation as follows:

$$\mathrm{Vol}_0 - \mathrm{Vol}_{\mathrm{fuel}} - \mathrm{Vol}_{\mathrm{eng}} = \mathrm{Vol}_{\mathrm{L.G.}} + \mathrm{Vol}_{\mathrm{pas}} + \mathrm{Vol}_{\mathrm{cockpit}} + \ldots \tag{14}$$

For this design study it is assumed that $\mathrm{Vol}_{\mathrm{nac}} - \mathrm{Vol}_{\mathrm{eng}}$ is small and can be neglected, based on examination of current airplane configurations. If the quantity on the right can be evaluated or shown to follow a trend, then a relation between fuel capacity and usable volume is possible. A plot of the quantity, $\mathrm{Vol}_0 - \mathrm{Vol}_{\mathrm{fuel}}$, against that on the right showed a direct relation of

$$\mathrm{Vol}_0 - \mathrm{Vol}_{\mathrm{fuel}} = 0.901\,\mathrm{Vol}_0 \tag{15}$$

The corresponding fuel weight, for jet fuel at a specific weight of 50.1 lb/ft^3, is, from Eq. (15):

$$W_{\mathrm{fuel}} = 5.01\,\mathrm{Vol}_0 \tag{16}$$

Now the task is to relate the total usable volume in terms of the same parameters as will be used in defining Eq. (7). From Ref. 1, the wing volume for normal taper ratios, constant wing thickness ratio, and most airfoil sections is approximated by

$$\mathrm{Vol}_W = 0.717\,(t/c)\,(S^{3/2}/\mathcal{R}^{1/2})$$

and the fuselage volume is approximated by

$$\mathrm{Vol}_{\mathrm{fus}} = 0.752 f F^{3/2}$$

Therefore the total usable volume is

$$\mathrm{Vol}_0 = 0.717\,(t/c)\,(S^{3/2}/\mathcal{R}^{1/2}) + 0.752 f F^{3/2} \tag{17}$$

Fuel requirements for other than cruise, such as takeoff, climb, descent, and 45-min reserve at destination, can be approximated, initially, by an allocation of 25% of the maximum internal fuel capacity. Thus

$$0.75\,W_{\mathrm{fuel}} = W_0 - W_F \tag{18}$$

but $W_F = A\,W_0$. Substituting in Eq. (16) gives

$$[(1-A)/0.75]\,W_0 = 5.01\,\mathrm{Vol}_0 \tag{19}$$

Structural Considerations

The number of variables can be reduced further if practical structural considerations are applied. Using a presently attainable spar aspect ratio of 50 results in a relation between aspect ratio and thickness ratio as follows:

$$\frac{\mathcal{R}}{(t/c)\cos\Lambda} = \frac{2}{1+\lambda} \times 50 \tag{20}$$

This relationship was developed and plotted in Ref. 1 as $\mathcal{R}/(t/c)\cos\Lambda$ against taper ratio λ. Values attained on present day jet airplanes were also plotted in the figure to show the trend. From inspection of the plotted data in Ref. 1, it was seen that present design trends yield values of $\mathcal{R}/(t/c)\cos\Lambda$ practically independent of taper ratio. Therefore a representative value of $\mathcal{R}/(t/c)\cos\Lambda$ (Ref. 1) is 55. (The values ranged from 32 to 96.)

To reduce the number of parameters in Eqs. (7) and (19) still further, there now appears a choice of eliminating either thickness ratio or aspect ratio. Reference 6 indicates that the parameters affecting the improvement in drag rise Mach number in order of decreasing effect are 1) sweep, 2) thickness ratio, and 3) aspect ratio. Since higher cruising speeds are a result of a higher drag rise Mach number and as this is one of the design requirements, the order of effectiveness given will govern the elimination of one of the variables, namely, the aspect ratio. Thus the form used will be

$$\mathcal{R} = 55\,(t/c)\cos\Lambda \tag{21}$$

for substitution in Eq. (7).

Another structural relationship is used to combine the fuselage fineness ratio, frontal area, and weight. This relationship, developed in Ref. 1, is

$$F = W_0/0.00137 f V_i^2 \tag{22}$$

where V_i was in mph. The final working version of Eq. (7) is now obtained by substituting the values of K, TSFC$_{36}$, and R in the proper places in Eqs. (8), (12), (21), and (23), and simplifying:

$$\frac{1-A}{\sqrt{(1+A+A^2)/3}} = \frac{1906.7}{V_c}\left[\frac{0.0153K_{TD}}{\sqrt{(t/c)\cos\Lambda}} + \frac{0.000406}{W_0}V_i^2 - \frac{0.0189}{f} + 0.00955\right] \tag{23}$$

This value of A will be called A_R.

The final working version of Eq. (19) is also obtained by substituting the relation $S = W/(W/S)$ in the proper places in Eqs. (18), (21), and (22), and simplifying:

$$1 - A = 3.76(\mathrm{Vol}_0/W_0) \tag{24}$$

where

$$\mathrm{Vol}_0 = W^{3/2}\left[\frac{0.0967}{(W/S)^{3/2}}\sqrt{\frac{t/c}{\cos\Lambda}} + \frac{9641}{f^{1/2}V_i^3}\right] \tag{25}$$

This value of A will be called A_V.

Airplane Performance Considerations

Because the takeoff distance is a design requirement, its relation to the size of the airplane must be considered. If the parameters in the Hartman takeoff equation[7] are considered,

$$S_g = 0.0384\,W V_u^2/T_e \tag{26}$$

where T_e is the net accelerating force at 0.7 of V_u, the unsticking speed in knots, and

$$V_u = \sqrt{\frac{295.1(W/S)}{C_{L_u}}}$$

it is seen that for optimum C_{L_u}, the takeoff lift coefficient, the takeoff ground distance is proportional to $W/T_0 \times W/S$ for $T_0 \approx T_e \gg (\mu W_0 + D)$, where μ is the ground friction coefficient. The total distance over a 35 ft height will also be proportional to $W/T_0 \times W/S$. Figure 4 provides the experimentally determined relationship for current business jet airplanes balanced field length.

The design requirements also set an approach speed. The landing configuration stalling speed, defined as the approach speed divided by 1.30, is a function of both the maximum lift coefficient and the wing loading. It is therefore necessary to establish what the attainable maximum lift coefficients are. Data from Ref. 1 present the maximum lift coefficient, with and without flaps and auxiliary high-lift devices, as a function of wing sweep in Fig. 5. It can be seen in Fig. 5 that the relation $C_{L\max}(\Lambda = 0 \text{ deg})\cos\Lambda$ approximates the effect of sweep on the maximum lift coefficient. The values of maximum lift coefficient obtainable for the thin wings agree closely with those derived from published airplane stalling speeds.

Cabin Volume

Whereas in Ref. 1 the analysis was concerned with only the pilot occupying space in the airplane, in the current study the fuselage must provide space for ten passengers, their baggage, and amenities such as a galley, tables, and a toilet. While business airplane buyers and salesmen point to the Falcon 20 or the Gulfstream 2 and 3 as the "cabin yardstick" for cabin comfort, no empirical parameters could be identified which measure this factor directly. The only design guideline to be applied is that the optimum design fall within the area of current design as defined by a plot of passenger cabin and baggage volume vs the fuselage volume in Fig. 6. The parameters of 18 current business jet airplanes were plotted in Fig. 6 to provide the limits of current design for cabin size.

Method of Calculation

The method of calculation is a graphical solution of the simultaneous Eqs. (23) and (24) for constant values of cruising speed and wing sweep, and with weight as the independent variable and A as the dependent variable. Intersection of the curves for Eqs. (23) and (24) indicates that the amount of fuel required to fly the desired mission with a particular configuration can be accommodated within that configuration. The design weight for any configuration, with a specified cruising speed and wing sweep, and the ratio of landing weight to design weight are obtained directly from the graphical solution. It is then necessary to tabulate these solutions and apply the conditions governing takeoff distance and approach speed. Without complicating matters by also considering stability limitations, it is possible from this tabulation or from a plot of this information against cruising speed to find the configuration which will give the highest cruising speed for the least takeoff weight.

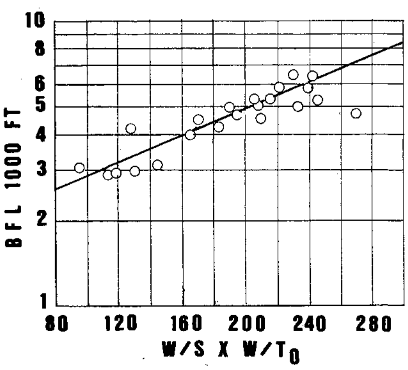

Fig. 4 Business jet airplane Federal Aviation Regulations (FAR) balanced field length relationship—sea level, dry runway.

Calculating Procedure

The following conditions are specified so that A_R and A_V can be calculated:

1) select best cruising speeds between 400 and 525 knots,
2) select sweep angles between 0 and 50 deg,
3) select design weights between 10,000 and 30,000 lb, and
4) set a cruising altitude of 41,000 ft.

The procedure for the actual calculations is as follows:

1) For each of the cruising speed values selected find the corresponding Mach number at 41,000 ft altitude. This Mach number will also be the drag rise Mach number of the fuselage and wings.

2) From Fig. 2, find the wing thickness ratio at the selected wing sweep angle that has the above drag rise Mach number.

3) From Fig. 3, find the fuselage fineness ratio that has the above drag rise Mach number, using the curve marked "M_{DR} with interference."

4) From items 1-3 above, all other required factors such as K_{TD}, $\cos\Lambda$, and q can be found.

5) For a number of assumed gross weights, and items 1-4 above, compute A_R by means of Eq. (23). (A graphical solution for A in terms of (TSFC)$R\phi/V$ and $\sqrt{3/(1+A+A^2)}$ can be prepared using Eqs. (3), (4), and (6) to rapidly solve for A.) Note that the right-hand side of Eq. (23) is (TSFC)$R\phi/V$). The variation of A_R, as the abscissa, with weight is then plotted.

6) For the conditions and items used in item 5 above, compute A_V by means of Eq. (24). Plot the variation of

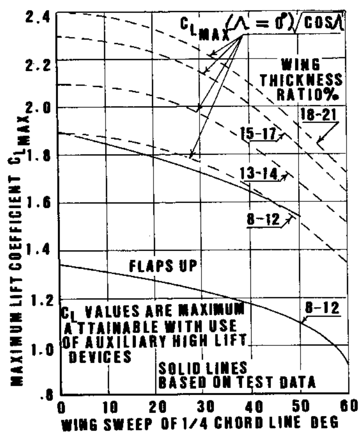

Fig. 5 Airplane maximum lift coefficient.

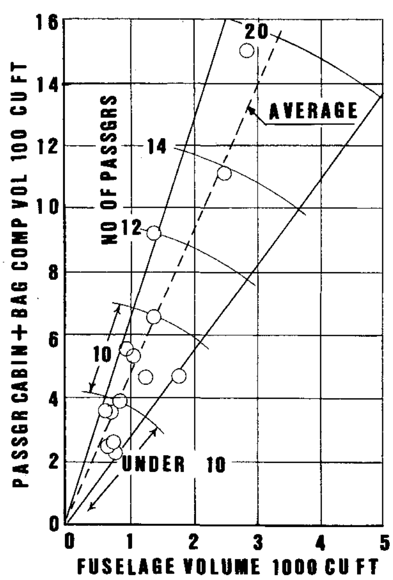

Fig. 6 Cabin volume as a function of fuselage volume and passenger number.

weight with A_V, as the abscissa, on the same figure as A_R. Additional intermediate weights can be selected and the values of A_V corresponding to them plotted in order to define the curve of Eq. (23) more precisely. A plot of A_V against A_R for constant sweep angles can also be used to define the intersection, which gives the required value of A.

7) The optimum wing loading based on the design gross weight is determined from

$$\left(\frac{W_0}{S}\right)_{\text{opt}} = q_c\sqrt{\frac{3}{1+A+A^2}}\sqrt{\frac{C_{D_{W+T}}}{K/\mathcal{R}}}$$

which is derived in Ref. 3. Substitution of Eqs. (8) and (21) and $K=1/0.8$ in the above equation results in

$$\left(\frac{W_0}{S}\right)_{\text{opt}} = 1.059K_{TD}q_c\sqrt{\frac{3}{1+A+A^2}}\sqrt{\frac{t}{c}}\cos\Lambda \tag{27}$$

8) The thrust required for any configuration is computed from

$$T_r = C_{D_{W+T}}qS + C_{D_F}qF + \frac{(K/\mathcal{R})\,W^2}{qS} \tag{28}$$

Substitution of Eqs. (8), (11), and $K=1/0.8\pi$ in the above equation results in

$$T_r = 0.0081K_{TD}^2 + \left(\frac{1.466}{F} + 0.011f - 0.0218\right)qF + \frac{W_0^2}{2.51\mathcal{R}qS} \tag{29}$$

At this stage we have defined the design weight (at the start of the cruise portion) and, by simple performance calculations, the gross weight at the end of the cruise portion can also be found. The fuel for holding at 30,000 ft for 45 min, which was originally included in the 25% fuel set aside, can now be computed.

9) Compute the fuel used in climb and the distance covered for this design weight at the cruising altitude.

10) Compute the fuel used in descent and distance covered.

11) The takeoff distance is estimated from Fig. 4 for the takeoff weight corresponding to the selected design weight at altitude, the computed wing areas, and the takeoff thrust.

12) The stalling speed is computed for the maximum landing weight and the maximum lift coefficient for the sweep angle selected and the derived wing thickness from Fig. 5. The maximum landing weight for each design weight and sweep was estimated as 0.89 of the design weight based on a survey of current business jet airplane designs.

13) Items such as the wing area, frontal area, fuel capacity, maximum fuselage diameter and length, and aspect ratio are obtained from the following basic relationships:

$$S = \frac{W_0}{W_0/S} \qquad b = \mathcal{R}\times S$$

$$\mathcal{R} = 55\,(t/c)\cos\Lambda \qquad d' = \sqrt{4F/\pi}$$

$$F = W_0/0.001818fV_i^2 \qquad l = f\times d'$$

14) A summary table containing the above design and performance items for several wing sweep configurations is prepared for each cruising speed. The various independent parameters, such as wingsweep, cruising speed, and gross weight, are then plotted against cruising speed as the abscissa. The optimum wing loading is plotted against design weight, as shown in Fig. 7.

15) The highest cruising speed is determined by the amount of thrust available. Thus the intersection of the maximum cruise thrust available with the thrust required curves for the various derived configurations will give the maximum cruising speed. For this cruising speed, the design weight, stalling speed, and value of A can be determined from the aforesaid plots.

Selection of the Optimum Configuration

Application of the summary figures described above to a specific performance requirement can now be carried out. As pointed out earlier, the airplane with the optimum configuration for long range must also meet the takeoff and stalling speed requirements. A graphical solution is obtained on a plot of the variation of optimum wing loading with design weight and sweep by plotting the gross weights and wing loadings which will give balanced field lengths of 5000 ft with takeoff thrust or $W/S = 202\ T_0/W$. A sample solution is shown on Fig. 7. A choice of wing planforms is available that meet the takeoff requirements. From a plot of stalling speed for the maximum landing weight from step 12, against best cruising speed for various sweep angles, select the airplane designs with wing planforms not exceeding a 100-knot stalling speed. Plot these corresponding "optimum" values of stalling speed and cruising speed as separate curves for airplane configurations with a 100-knot stalling speed and those with a 5000-ft balanced field length (BFL) capability against the configuration wing sweep angle for one solution. Then plot the corresponding "optimum" values of design weight and thrust required as separate curves for the same airplane configurations (100 knot stalling speed and 5000 ft BFL) also against configuration wing sweep angle for the second

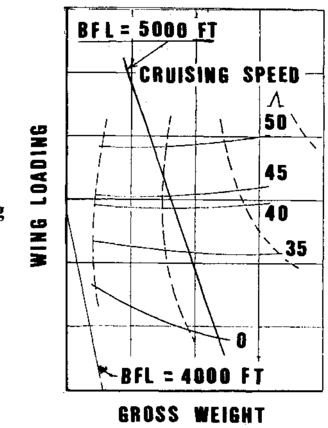

Fig. 7 Variation of wing loading with gross weight.

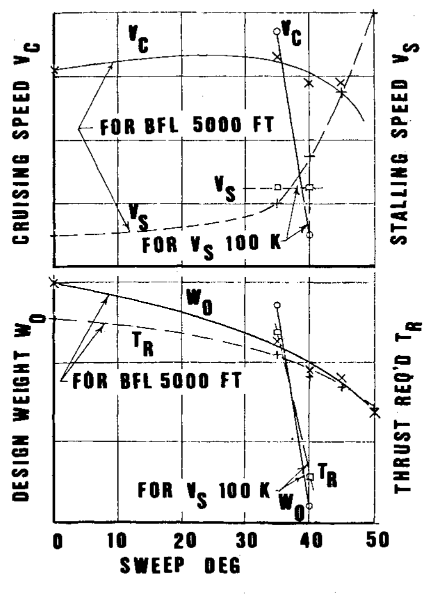

Fig. 8 Determination of optimum configuration.

Table 2 General particulars for the optimum configuration

Ten-passenger turbofan airplane	
Maximum ramp weight	21,575 lb
Maximum landing weight	19,202
Empty operating weight	12,367
Fuel capacity	1136 gal, 7608 lb
Wing area	294.7 ft^2
Wing span	43.8 ft
Aspect ratio	6.51
Section thickness	14.6%
Sweep $c/4$ chord	36 deg
Volume of passenger cabin and baggage compartment	520 ft^3
Fuselage, maximum	
Frontal area	40.4 ft^2
Diameter	7.17 ft
Length	39.9
Fineness ratio	5.56
Maximum rate of climb, sea level	4329 ft/min
Balanced field length	5300 ft
Stalling speed in landing configuration	98.5 knots
Approach speed in landing configuration	128.1
Maximum range at 41,000 ft with 45-min fuel reserve (ten passengers)	2500 n.mi.
Maximum cruise speed at 41,000 ft	465 knots ($M = 0.81$)

solution. The intersection of each pair of curves defining a design parameter for both design conditions defines the airplane configuration that meets all the performance requirements originally spelled out. A sample solution is shown in Fig. 8.

Results

Particulars of the design with the optimum configuration are presented in Table 2. Some performance features for this design were computed to show its possibilities and capabilities using estimated drag characteristics presented previously and the thrust characteristics for the uninstalled GA TFE 731-3 engines. The results are also shown in Table 2.

Discussion

The design as developed in this study is compared briefly with the Cessna Citation III and the Rockwell International Sabreliner 65 in dimensions and performance to show that the analytical approach and data used provide realistic results. Both current airplanes bracket the optimum design in weight categories, balanced field length, maximum sea level two-engine rate of climb, fuel capacity, high-speed cruise, wing section thickness, and cabin volume. They differ in approach speed, fuselage fineness ratio, wing area, span and sweep, aspect ratio, fuselage maximum frontal area, and maximum range. The optimum configuration has a longer range with 10 passengers than either the Citation III with 10 passengers or the Sabreliner 65 with 4 passengers. The optimum design has a 36-deg wing sweep and a 14.6% thickness airfoil section, while the Citation III has a 25-deg wing sweep and a 19% thickness airfoil, and the Sabreliner 65 has a 29-deg wing sweep and an 11% airfoil section. The design in Ref. 1 for a long-range jet fighter also compared favorably with a long-range Navy attack airplane in its configuration, and indicated that for subsonic designs it also provided realistic results.

In addition to the optimum design, other design guidelines can be generalized. Analysis of data generated showed that to obtain cruising speeds above 462 knots, sweep angles greater than 36 deg with a wing thickness ratio of less than 14.6% are required; however, the stalling speeds will also increase and exceed 104 knots. To maintain the low stalling speed with the higher sweep angles and thin wings requires sophisticated high-lift devices on the wings. From an examination of Eqs. (23) and (24) it is seen that if the drag rise Mach number for either the fuselage or wing is increased, resulting in higher fineness ratios or lower thickness ratios, respectively, the value of A_R decreased and the value of A_V increased. This means that more fuel is required to fly the required range, because the fuselage drag has increased by Eq. (11) or the wing induced drag has increased by Eq. (28) due to a lower ratio, defined by the relationship $\mathcal{R} = (t/c)\cos\Lambda$. Meanwhile, the airplane usable volume has decreased because the fuselage cross-sectional area decreased at a greater rate than the fuselage length increased by Eq. (22). As a result, if a solution is possible, it would be necessary to increase the design weight above that for the optimum configuration.

Another observation from studying the data compiled for Ref. 1 and for this study is that, at a constant cruise speed, as the sweep angle increases, the aspect ratio increases. The question of whether the volume defined by Eq. (24) can predict the amount of fuel that can be accommodated with certain fuselage dimensions was answered in Ref. 1 and showed that the average error is about -11%; i.e., A_V, as computed, is 11% low. A considerable amount of data in this paper can be used with the method outlined to develop an airplane configuration to meet other specific performance requirements.

Conclusions

The optimum configuration of a 10-passenger business turbofan airplane to meet specific performance requirements was developed using an analytical method. The method used to determine the configuration is based, wherever possible, on current design practice and gives results which are comparable with current airplane designs meeting similar performance requirements. In addition, the method and a number of the curves can be used to determine configurations which will meet other specific performance requirements.

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